A Walkthrough Solution to the Boundary Overlap Problem

Mark J. Ducey, Jeffrey H. Gove, and Harry T. Valentine

ABSTRACT. Existing methods for eliminating bias due to boundary overlap suffer some disadvantages in practical use, including the need to work outside the tract, restrictions on the kinds of boundaries to which they are applicable, and the possibility of significantly increased variance as a price for unbiasedness. We propose a new walkthrough method for reducing boundary overlap bias that diminishes or eliminates the need to work outside the tract and accommodates irregular boundaries easily. Under typical conditions, the walkthrough method eliminates the boundary overlap bias associated with most objects near the border and reduces it for the remaining objects. The walkthrough method is object-centered in conception and implementation, but the measurements required are simple. The walkthrough method complements existing methods for correcting boundary overlap bias and should prove especially helpful when conditions make existing methods difficult or impossible to use. FOR. SCI. 50(4):427–435.

Key Words: Forest sampling, mirage, reflection, relascope, prism cruising.

THE POTENTIAL FOR BIAS DUE TO BOUNDARY OVERLAP exists in nearly all types of forest sampling. The problem arises whenever objects (trees, downed logs, etc.) within the population being sampled lie close to the boundary of the tract. Ordinarily, an object is measured if a sample point falls within that object's inclusion zone. If the inclusion zone extends beyond the tract boundary, then that object will have less than its nominal probability of being selected. Objects near the boundary will thus be underrepresented (in expectation) in the sample, and the usual estimators of attributes like density, basal area, and volume will be biased downward. As Gregoire (1982) has pointed out, bias occurs whether or not sample points actually fall near the boundary; because bias is a matter of expectation, it occurs whenever objects are sufficiently near the boundary to cause boundary overlap.

A number of useful methods for correcting bias due to boundary overlap have appeared in the literature or are used in practice (see, e.g., Schreuder et al. 1993 pp. 297–301). Of these, the most widely discussed in textbooks is the mirage method introduced by Schmid-Haas (1969). However, the inability of the mirage method to deal with irregular boundaries, and with linear or pocket inclusions such as roads or landings, presents a severe limitation to its use in some settings (Iles 2000). The common practice of either rejecting sample points that fall near the boundary or moving such points away from the boundary can be detrimental and lead to bias (Gregoire and Scott 2003). The continued widespread use of this practice testifies at least in part to the perceived difficulty or impracticality of existing unbiased boundary correction methods.

In an effort to deal with the perceived shortcomings of

Mark J. Ducey, Associate Professor of Forest Biometrics, Department of Natural Resources, University of New Hampshire, Durham, NH 03824—Phone: (603) 862-4429; Fax (603) 862-4976; mjducey@cisunix.unh.edu. Jeffrey H. Gove, Research Forester, USDA Forest Service, Northeastern Research Station, PO Box 640, Durham, NH 03824—Phone: (603) 868-7667; jgove@fs.fed.us. Harry T. Valentine, Research Forester, USDA Forest Service, Northeastern Research Station, PO Box 640, Durham, NH 03824—Phone: (603) 868-7671; hvalentine@fs.fed.us.

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these methods, we have developed an alternative, based on a modification to the boundary reflection method of Gove et al. (1999). The method is object-centered in its conception and implementation and therefore compatible with the object-centered development of the boundary overlap problem presented by Gregoire (1982). It can be implemented fairly rapidly in the field, without predetermining quantities such as the largest object expected on the tract. It also deals with irregular boundaries: the modified boundary reflection method is unbiased across a broad range of boundary configurations and it reduces bias in the others. Before introducing the method, however, we should briefly examine two popular methods for contending with boundary overlap and consider their advantages and disadvantages.

Sampling Outside the Tract Boundary

One method that is unequivocally design-unbiased is to allow sample points to fall outside the tract boundary, but within some region of known area that completely encompasses all possible inclusion zones for objects in the tract (Figure 1). Only objects in the tract are tallied; objects outside the tract but within the encompassing region are ignored. If estimates are obtained on a per hectare or per acre basis, they are first expanded by the area of the encompassing region, then divided by the tract area to obtain unbiased estimates. This method was first developed by Masuyama (1954) in an agricultural context and discussed briefly in a forestry context by Schmid-Haas (1982) and by Mandallaz (1991), who indicated "This is the only simple and exact universal method. . . to construct unbiased estimates." Iles (2000, 2001) presents a related "toss-back" method that has similar features. The name "toss-back" is motivated by the idea that both the tally of any boundary trees, and the portion of the inclusion zone for all boundary trees, is "tossed back" onto points in the tract.

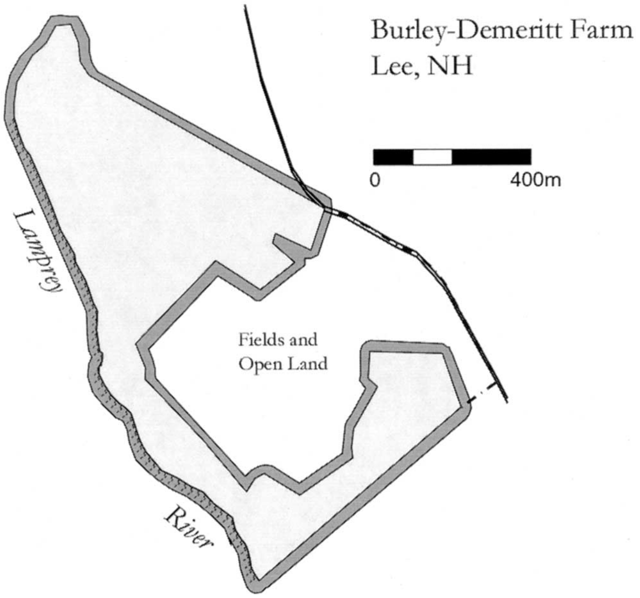

Figure 1. Sampling outside the tract, illustrated for a farm woodlot (light gray, 46.5 ha) in southern New Hampshire. In addition to the actual woodlot, plots would be allowed to fall into a 25.1-m buffer area (dark gray) to encompass all possible inclusion zones (assuming a 2.3-m^2/ha prism and a 76-cm maximum tree size). The total area in which samples could fall increases to 60.7 ha, or a 30% increase. Unfortunately, part of the buffer area falls into the Lamprey River. The method is unbiased but, in this case, impractical.

The principal advantage of this method is that it is design-unbiased for any boundary configuration and any inclusion zone shape. However, it presents some practical difficulties:

1. The boundaries of the encompassing region must be determined in advance, or a rule for determining the boundaries must be predefined. This is most often accomplished by establishing a buffer of fixed width around the original tract; sample points falling within the buffer are measured.
2. The boundaries of the encompassing region must be determinable in the field. This may present some difficulties when boundaries are quite irregular.
3. In probability proportional to size sampling, these methods require the cruiser to stipulate in advance the maximum size object that may be encountered on the tract. More specifically, it must be possible to determine the maximum size inclusion zone for any object and then buffer the tract boundary so that all possible inclusion zones for objects at the tract boundary are encompassed by the buffer.
4. In Masuyama's (1954) approach, the area of the encompassing region must be computable. This requirement is straightforward if geographic information system technology is available and the original tract boundary has been digitized. However, in many applications the technology or data are not available, or the cost of using them is not justified. Computing the area of the encompassing region is not required using the toss-back method (Iles 2001).
5. Finally, the cruiser must be able to travel and work accurately outside the original tract boundary. This may be impossible when the boundary is set by terrain features such as cliffs, bodies of water, or highways. It may be impractical when the boundary is shared by an adjoining hostile owner, or even the lawn of a private residence.

Furthermore, while these methods are unbiased, their performance relative to other measures such as mean squared error is suspect, especially when time efficiency is considered. Intuitively, the inclusion of a random number of sample points that are empty or nearly empty (relative to those taken inside the original tract boundary) seems certain to inflate the sample variance, in comparison to approaches that restrict sampling to the tract interior. For tracts or strata that are particularly "edgy," for example riparian stands with many large trees, the encompassing region may have to be considerably larger than the original tract or stratum. This will prove especially true when inclusion zones for large objects may be of significant size, for example in horizontal line sampling (Strand 1957, Beers and Miller 1976) or related “sausage sampling” methods (Ducey et al. 2002). Spending time to sample an area larger than the tract of interest, and obtaining an estimate that may have a substantially inflated variance, but eliminating boundary overlap bias, may seem like a poor bargain in many applications.

Mirage Method

The mirage method has been widely advocated as a practical approach to eliminating boundary overlap bias (Beers 1977). Since its original development by Schmid-Haas (1969) for fixed plot and horizontal point sampling, its unbiasedness has been proven (Gregoire 1982), and it has been extended to line intersect sampling (Gregoire and Monkevich 1994). It is recommended in most major introductory and intermediate mensuration textbooks (Loetsch et al. 1973 pp. 326–327, Zöhrer 1980 pp. 28–29, Husch et al. 1982 pp. 274–275, de Vries 1986 pp. 232–233, Shiver and Borders 1996 pp. 72–73, Avery and Burkhart 2001 pp. 241–242) and is therefore accessible to a wide range of practitioners.

In the mirage method, whenever a sample point (e.g., plot center in fixed plot sampling or point for angle-gauge sampling) is located close to the boundary, a mirage point is installed. The mirage point is located by reflecting the original sample point through the adjoining boundary, as shown in Figure 2. The sample method is performed again at this mirage point, and only objects inside the tract are tallied. The tally from the mirage point and the original point are added together. Variants of the mirage method specialized to rectangular plots (Schmid-Haas 1982) and line-intersect sampling (Gregoire and Monkevich 1994) have also been proposed. Unbiasedness of the mirage method derives from the reflection or folding over of the inclusion zones for boundary trees back into the tract.

While the mirage method is extremely straightforward to implement under many field conditions, it also suffers notable drawbacks:

1. The method is only unbiased when the relevant portion of the boundary is linear within its intersection with every inclusion zone, or when restrictive conditions on the angles of corners apply.
2. When boundaries are irregular, and in the face of linear inclusions (such as roads) or pocket inclusions (such as landings) the method as classically considered fails entirely (Iles 2000).
3. When inclusion zones are not circular, some trees tallied from the mirage point may not be tallied from the original point. Thus, it may even be necessary to install a mirage point when the original point resulted in an empty tally. Field crews who do not fully understand the geometry of the method may fail to implement it correctly, leading to bias in application (Gove et al. 1999, Ducey et al. 2001).
4. Travel outside the tract is required to establish the mirage point. As noted above, terrain or other factors may render this difficult, dangerous, expensive, or impossible.

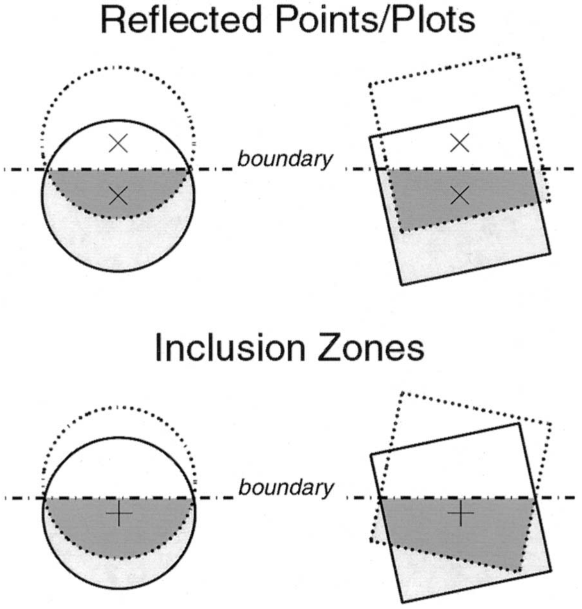

Figure 2. The mirage method for circular and square inclusion zones (following Schmid-Haas 1969). The top panel illustrates the application of the method, in which a mirage point is located by reflecting the original sample point through the boundary. A new sample of objects inside the boundary is taken from the mirage point, and the tally is added to the original tally. The bottom panel shows the effect on inclusion zones of the objects. The area enclosed by the dotted line (“mirage inclusion zone”) indicates the zone within which the original point could fall, and the object (+) would be tallied from the mirage point. If the original sample point falls in the light gray area, the object is tallied once; if it falls in the dark gray area, the object is tallied twice. Because the area of the mirage inclusion zone is identical to the area of the original inclusion zone that fell outside the tract, the method is unbiased (Gregoire 1982).

Both the toss-back method and the mirage method have attractive features, but because both methods also require sampling outside the boundary, they are inapplicable in certain situations. A method that could contend with a broader array of boundary conditions than the mirage method, yet be simple to implement in the field, and not require travel and sampling outside the tract boundary, could represent an attractive complement to these two sets of techniques.

Boundary Reflection Method

As a starting point in developing a method with attractive features, we reexamined the boundary reflection method originally proposed by Gove et al. (1999) in the context of sampling downed coarse woody material. Although the mirage method is also sometimes called the reflection method, the two approaches are different. The boundary reflection method is invoked whenever a tallied object appears close to the boundary of the tract. When this occurs, the field crew reflects the boundary about the object, as illustrated in Figure 3. If the object is a line, then the boundary is reflected geometrically through the line defined

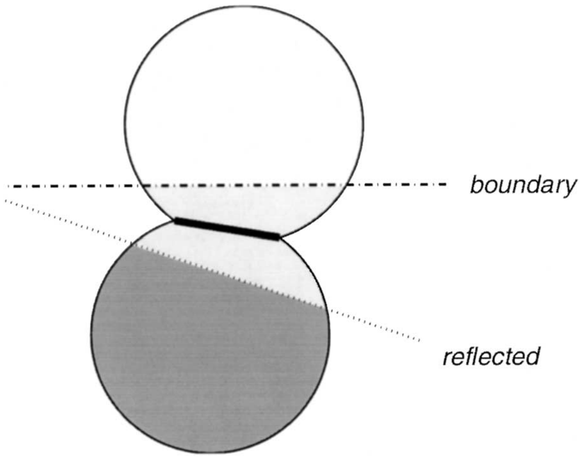

Figure 3. The boundary reflection method of Gove et al. (1999) as developed for point-relascope sampling of downed logs. The boundary is reflected through the tallied log (solid line), and if the reflected boundary falls between the sample point and the log, the object is double-tallied. The area where the sample point would yield a double tally, shown as dark gray, equals the portion of the inclusion zone that falls outside the tract. Therefore, the method is unbiased.

by the object. If the object is a point, then the boundary is reflected geometrically through that point. In either case, if the reflected boundary falls between the sample point and the object, the object is tallied twice. Gove et al. (1999) present a proof of unbiasedness in the case of downed coarse woody material sampling with straight-line boundaries that extends readily to other types of sampling, including fixed-plot and angle-gauge sampling.

Figure 4 shows the boundary reflection method as applied to point-relascope sampling of coarse woody material, sampling standing trees with a circular inclusion zone (as would arise using fixed circular plots or with an angle gauge), and sampling with a square inclusion zone. It is clear that the unbiasedness of the boundary reflection method extends to include not only a variety of inclusion zone shapes, but also a variety of boundary configurations. It does not require travel outside the tract boundary, as the fieldwork can be completed entirely within the boundary of the tract. Furthermore, it has the added advantage that all "double-tallied" objects must have been included in the original tally, regardless of inclusion zone shape. However, it is equally clear that for an arbitrarily complex boundary, reflecting that boundary through the object may require an arbitrarily large expenditure of field effort.

Walkthrough Method

A straightforward modification of the boundary reflection method, motivated by a simple insight, suffices to remedy this last problem. In the boundary reflection method, the boundary is reflected through the object, and the position of the reflected boundary is assessed relative to the sample point. This is geometrically equivalent to reflecting the sample point through the object and comparing its

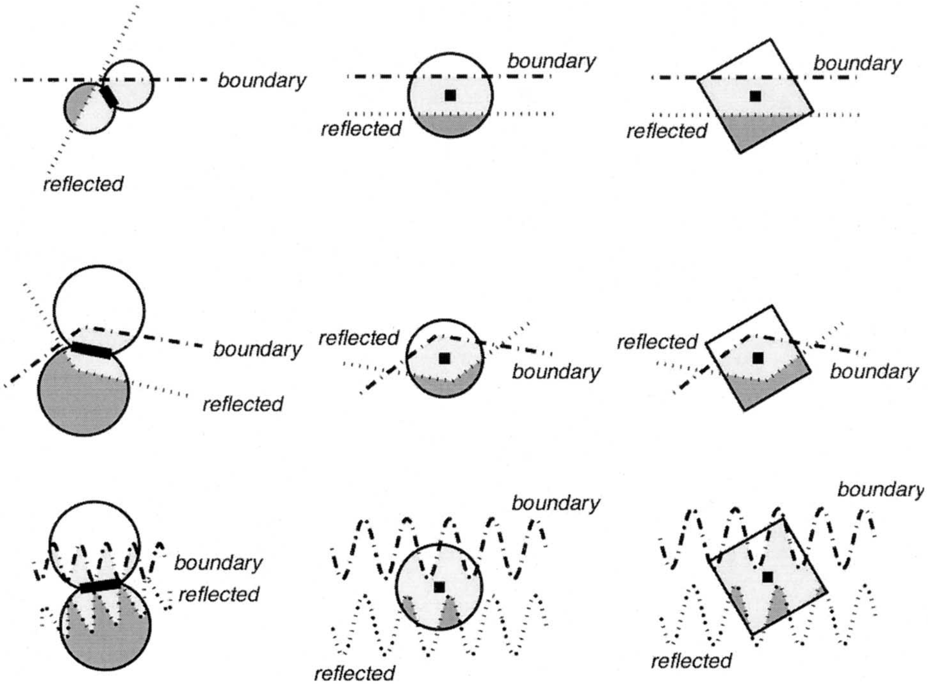

Figure 4. The boundary reflection method applied to point-relascope sampling of coarse woody material (first column), circular inclusion zones (second column), and square inclusion zones (third column), with a variety of boundary configurations. In all cases shown, if the sample point falls in the dark gray area, the reflected boundary will fall between the sample point and the object, and a double tally will result. Because the double-tally area equals the portion of the inclusion zone that falls outside the tract, the method is unbiased for these cases.

position with the boundary. This latter procedure is operationally much simpler to implement, because it requires reflecting only a single point, not an entire section of boundary. Furthermore, in the case of linear objects such as downed logs, it is immaterial to the theoretical properties of the method whether reflection is through the line defined by the object, as in Figure 3, or simply through the point at the center of the object. (Strictly speaking, this center point should lie at the center of the inclusion zone of the object; it need not be the physical center of the object.) When sampling linear objects such as downed logs, reflection through the center of the object simplifies the required measurements, and we recommend this approach in using the modified method. For reasons that will become obvious in the practical description below, we call this modified method the walkthrough method. We take a graphical approach to demonstrating unbiasedness here; formal symbolic proofs and conditions for unbiasedness of the walkthrough method, using Horvitz-Thompson (1952) and Monte Carlo (Valentine et al. 2001) approaches, are available from the authors.

Table 1. A decision key for field implementation of the walkthrough method. The key is entered whenever a tallied object appears close to the boundary.

I. Is it possible that the tallied object is closer to the boundary, than to the sample point?
  Ia. NO—No action needed. Tally the object normally.
  Ib. YES—Proceed to II.
II. Measure the distance from the sample point to the object—call this distance *x*. Now measure the distance from the object to the boundary, continuing on the same bearing. Call this distance *y*. Is *y* less than *x*?
  IIa NO—No action needed. Tally the object normally.
  IIb. YES—Proceed to III.
III. Does the boundary curve back across the walkthrough line?
  IIIa. NO—Walkthrough point must be outside the tract. Double-tally the object.
  IIIb. YES—Proceed to IV.
IV. Move to the walkthrough point, so that the distance to the object equals the previously measured distance *x* along the same bearing, or to a point where that location can be clearly identified. Is the walkthrough point inside the tract?
  IVa. NO—Double-tally the object.
  IVb. YES—Tally the object normally.

Implementation of this method uses the following protocol. When a tallied object is close to the boundary of the tract, in the sense that it may be closer to the boundary than to the sample point, a walkthrough is conducted. The first step in the walkthrough is to measure the distance and direction from the sample point to the center of the object. (Under good visibility conditions, and if the sample point is well marked, a direction measurement may not be needed; a distance measurement may even be unnecessary if the position of the object relative to the boundary is clear.) The forester then “walks through” the same distance and direction past the center of the object. If the point so located falls outside the tract, the object is tallied twice. Note that in all cases except highly curved or irregular boundaries, encountering the boundary will be sufficient to demonstrate that the walkthrough point lies outside the tract; in most cases, travel outside the tract is not required. Specific field implementation of the method is described using a decision key in Table 1 and illustrated graphically in Figure 5. Inclusion zones and the areas in which the tally is doubled for different objects are shown in Figure 6. For the examples shown in Figures 4 and 6, the aggregate areas are the same whether boundary reflection or walkthrough is used, though their orientation may change if the definition of reflection through the object changes. Because the difference between the walkthrough and boundary reflection methods is purely operational, the walkthrough method is unbiased whenever the boundary reflection method is unbiased.

The walkthrough method, like boundary reflection, is object-centered both in its theory and implementation. From one standpoint, this can be viewed as a disadvantage, in that measurements must be performed for every object tallied between the sample point and the tract boundary, that may

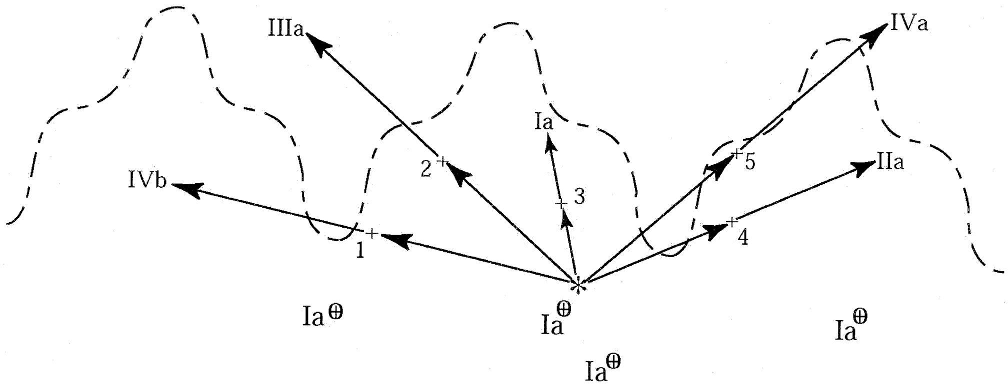

Figure 5. Graphic illustration of the walkthrough method. Five sample objects (+), lying close to the boundary, have been tallied from a sample point (*). The arrows indicate the layout of the walkthrough points for each object; the outcome on the key in Table 1 is indicated for each walkthrough point. Objects 1, 3, and 4 are tallied normally; objects 2 and 5 are double-tallied. Four objects (⊕) lie “close to the boundary” but in positions where they would be single-tallied, and no measurements would be needed.

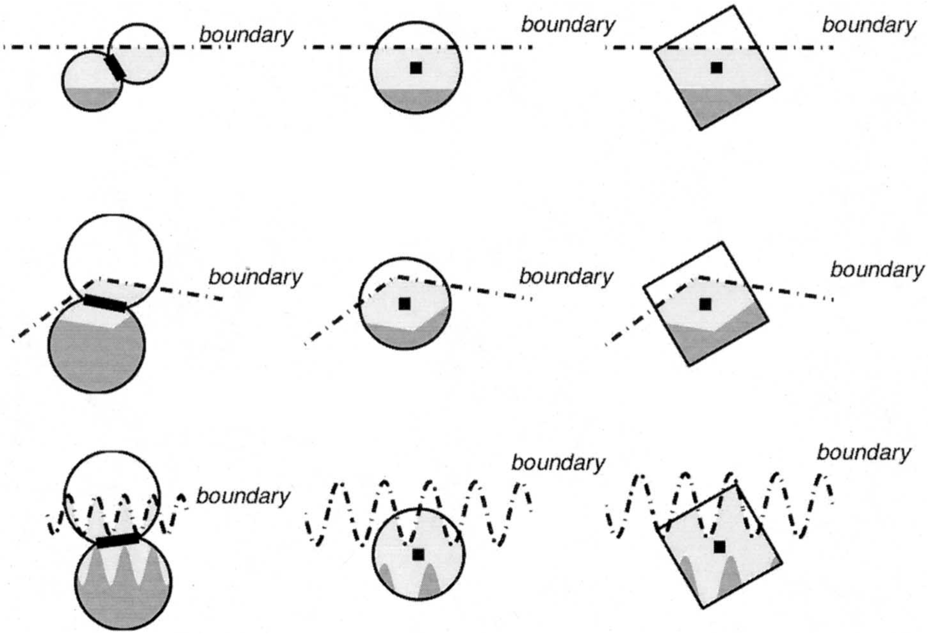

Figure 6. The walkthrough method applied to point-relascope sampling of coarse woody material (first column), circular inclusion zones (second column), and square inclusion zones (third column), with a variety of boundary configurations. In all cases shown, the area where the sample point would yield a double tally, shown in dark gray, equals the portion of the inclusion zone that falls outside the tract. Therefore, the method is unbiased.

be closer to the border than to the sample point. On the other hand, the number of objects meeting this condition is likely to be small, and the measurements required for the walkthrough method are extremely simple, unlike some other object-centered methods in the literature. In particular, the measurements required for the enlarged-tree circle method of Barrett (1964), and for the tree-concentric method described by Schreuder et al. (1993), appear problematic when the boundary is irregular. The measurement effort required to establish one walkthrough point for each tallied object close to the boundary does not depend substantially on boundary complexity. An advantage of object-centered methods, furthermore, is that they focus attention on the true source of boundary overlap bias: objects near the border (Gregoire 1982), not sample points near the border (Finney and Palca 1948).

The walkthrough method does not completely avoid the need to travel outside the boundary when the boundary is (1) highly convoluted (i.e., when the line between the object and the walkthrough point may pass out of the tract and back into the tract again), and (2) either topographic conditions or the methods of distance measurement used require straight-line travel to establish the position of the walkthrough point. However, in many instances the need to travel outside the boundary will be eliminated, and in others it will be greatly reduced. In addition, it enjoys the two principal advantages of the boundary reflection method: tolerance of a wide variety of boundary types, including irregular boundaries, linear inclusions, and pocket inclusions; and the fact that (by definition) only objects tallied from the original sample point can receive augmented tallies, regardless of inclusion zone shape. These advantages should make the walkthrough method attractive when sampling outside the boundary, or the mirage method, are disadvantageous.

Bias in the Walkthrough Method

While it has strengths, the walkthrough method also has shortcomings. We may state the formal requirement for unbiasedness of the walkthrough method as follows. For every point falling inside the inclusion zone of an object and outside the tract, the walkthrough point (i.e., the point located by reflection through the center of the object) corresponding to that point must fall inside the inclusion zone of the object and inside the tract. It is a necessary and sufficient condition for unbiasedness that this statement hold true for all objects in the tract, whether tallied or not. This condition may fail in one of two ways.

When inclusion zones are asymmetrical about the center of the object, a potential sample point may fall within the inclusion zone and outside the tract, but its walkthrough point may not fall within the inclusion zone (Figure 7). Fortunately, asymmetric inclusion zones are rare in forestry. The most common example is probably the use of fixed-area plots when the corner of a plot, instead of the plot center, is randomly located within the tract. Another example is the use of only half an angle-gauge sweep in horizontal point sampling, to reduce the expected tally (Iles and Wilson 1988). In line-based methods, such as line-intersect sampling and horizontal line sampling, if the end rather than the center of the line were located at random within the tract, inclusion zones would also be asymmetric. Fortunately, these situations are the exception rather than the rule. Furthermore, for all these examples except the method

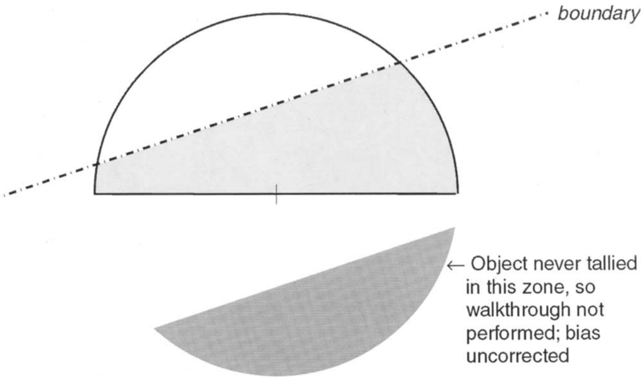

Figure 7. The walkthrough method may be biased with asymmetric inclusion zones, because a portion of the inclusion zone that falls outside the tract may not have a corresponding portion inside the original inclusion zone. This is shown here with a hemispherical inclusion zone, as in the method of Iles and Wilson (1988). The required double-tally area (dark gray) falls outside the inclusion zone, so the object is not tallied in the first place. The bias remains uncorrected.

described by Iles and Wilson (1988), if use of the walkthrough method is anticipated, it is easy to correct the problem by making the sample point the center of the plot or line during the design phase of the inventory. In the half-sweep method addressed by Iles and Wilson (1988), random selection of the half-sweep is required to avoid bias.

A more problematic violation of the conditions for unbiasedness, and one which is largely unavoidable, is illustrated in Figure 8. When an object is located relative to the boundary such that both a potential sample point inside the inclusion zone and its walkthrough point fall outside the boundary, some bias remains. It is not possible to completely eliminate the possibility of objects tucked into corners of a tract, or falling into long narrow corridors, by design. These difficulties—as with boundary overlap bias itself—can be reduced by using sample methods that induce compact inclusion zones. Note, furthermore, that even in cases such as Figure 8, the "folding over" of any excluded portion of the inclusion zone back into the tract reduces the sampling bias.

Sources of bias such as that depicted in Figure 8 cannot be eliminated completely in the walkthrough method, no matter what shape inclusion zone is used, though simulation results (Appendix I) suggest the residual bias may be very small in practical circumstances. Note that because the difference between the walkthrough and boundary reflection methods is primarily operational, these same situations also lead to bias in the boundary reflection method. Because boundary overlap bias cannot be guaranteed to be eliminated by design, we call the walkthrough method and the boundary reflection method bias reduction methods, rather than bias elimination methods.

Conclusions and Recommendations

As Iles (2001) points out, many sampling methods used in forestry must tolerate small biases to remain practical. At the same time, it is important that estimates derived using those methods be reasonably accurate, and it is important to reduce bias whenever it is cost-effective, so estimates will be defensible. As the simulation results of Gregoire and Scott (1990) indicate for horizontal point sampling, methods for correcting the bias due to boundary overlap may increase variance more than they decrease the bias square, so they may be deleterious to error measures such as mean square error if sample size is held fixed. Any increase in sample size required to offset increases in sample variance should be considered part of the cost of bias correction, in addition to the effort required to implement the method at each sample point.

All three of the methods discussed above present attractive and unattractive features in specific contexts. Based on those features, we suggest the following principles should guide selection of a correction method:

1. Where it is applicable (i.e., in parcels delimited by piecewise-straight boundaries with regular corners, and where working outside the boundaries is easy), the ease of use and familiarity of the mirage method make it an attractive option.

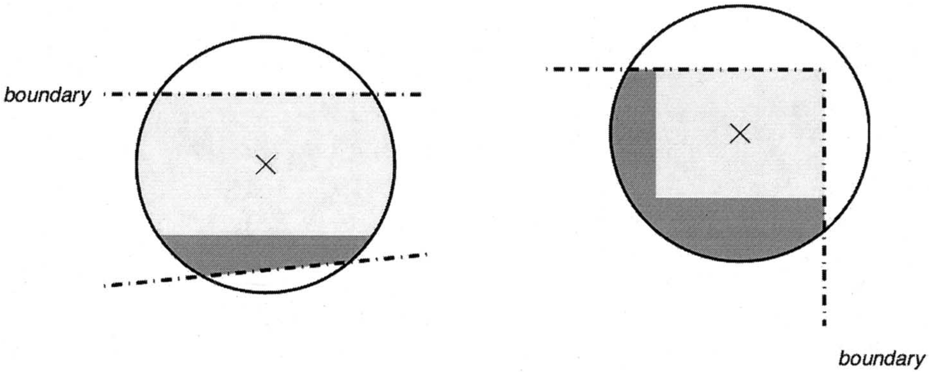

Figure 8. The walkthrough method will be biased whenever the boundary "wraps around" an object and within its inclusion zone. The double-tally area (dark gray) is less than the portion of the inclusion zone outside the tract, so the bias is not fully corrected for these objects. However, unless the boundary is perfectly symmetrical around the object, boundary overlap bias will still be reduced relative to the no-correction case.

2. When boundaries are irregular, but working outside the boundaries is easy, and where absolute unbiasedness is important to judging the results of a cruise, use either Masuyama's (1954) approach or the toss-back method proposed by Iles (2001).
3. When boundaries are irregular and working outside the boundaries is difficult or impossible, and where a small bias is preferable to the possibility of a significantly inflated variance due to the inclusion of numerous "empty" sample points, consider the walkthrough method proposed here as a practical alternative for bias reduction.

Finally, we suggest that further attention is needed to quantify the impact of both boundary "slopover" and the various proposed correction methods on both bias and variance in estimates of common quantities. Much of the work on boundary overlap to date has focused on theoretical efforts to attain absolute unbiasedness, without sufficient attention to the impact on sample variance or to field effort. In this vein, further work such as that by Gregoire and Scott (1990), possibly augmented by field trials, would be welcome in elucidating which boundary correction methods will be preferable in specific, well-defined circumstances.

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Appendix. Simulation Results

How important is the residual bias in the walkthrough method, as illustrated by Figure 8, in practice? As an illustration, consider again the tract depicted in Figure 1. We evaluated the magnitude of the expected undertally using a model-based, Monte Carlo integration approach. We assumed circular inclusion zones of 25.1-m radius centered on randomly located trees within the tract. Technically, this limiting distance connotes sampling with a fixed-radius plot, but it can also be interpreted as the limiting distance when sampling large trees for this region with horizontal point sampling using a typical but relatively small basal area factor (BAF), which should lead to significant boundary overlap bias and also to bias problems with the walkthrough method, if they exist.

The potential for bias using (1) no correction, (2) the mirage correction only where applicable, and (3) the walkthrough method was evaluated by Monte Carlo integration over this space of possible inclusion zones. For each iteration, a random coordinate within the tract was generated, and taken to represent the tree at the center of the inclusion zone. Then, a second random point was generated uniformly within the inclusion zone. The second random point represented a sampling location. If the sampling location fell within the tract, it represented a sampling location for which no correction was needed. If the sampling location fell outside the tract, it represented a sampling location for which correction was needed; i.e., a portion of the inclusion zone of the tree fell outside the tract, indicating a possible undertally and bias. By reflecting the sampling location through the tree location, and determining if the reflected point fell inside the tract, we could determine whether the walkthrough method would correct or fail to correct the missing tally corresponding to the sample point.

Monte Carlo integration, using 250,000 iterations, indicated an expected undertally of 6.7% when no boundary correction was used. A total of 32.8% of the tract boundary was taken up by features, such as the river, where the mirage method could not be employed. Employing the mirage correction where it was possible (but nowhere else) would reduce the expected undertally to 2.2%. Undertally using the walkthrough method was only 0.11%. All digits given here are significant digits given the Monte Carlo sampling error. While by no means exhaustive, and not necessarily reflective of all situations that may be encountered in practice, these results suggest that the walkthrough method can reduce bias substantially and that the residual bias, even for large inclusion zones on tracts of complex shape, may be relatively small.